A Multivariable Nonlinear Self-Tuning Controller

A new self-tuning controller (STC) has been developed for a general class of nonlinear multiinput, multioutput process models. These models can include arbitrary, known nonlinear functions of the old inputs and outputs as well as the products of these functions and any powers of the most recent inputs. A very general control algorithm has been proposed that allows different numbers of inputs and outputs, different time delays for each input-output pair, and a performance index that can include different penalties on the outputs and on the incremental changes in the outputs. Simulation results are presented for two-point composition control of a pilot-scale distillation column. The results demonstrate that the new nonlinear STC performs significantly better than conventional STC's based on linear models.

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Introduction

Although many real-life processes exhibit significantly nonlinear behavior around their nominal operating points, control systems based on linear process models often suffice for small changes in the operating conditions. In particular, self-tuning control systems (STC's) have been especially successful over a wide range of operating conditions by continually modifying linear models to fit new operating conditions (Åström and Wittenmark, 1973; Clarke and Gawthrop, 1975, 1979; Koivo, 1980; Seborg et al., 1986). Self-tuning control systems recursively estimate the parameters in an assumed dynamic model from input-output data at each sampling instant, and then update the controller settings based on the current parameter estimates. Thus, the adaption of a linear model to changing operating conditions is an inherently gradual process that may require a number of samples close to the new operating point before yielding a new linear model that would perform well at the new conditions. This limitation renders linear STC's far from optimal for large, rapid changes in the operating point of the process.

In practical applications, the nature of the process nonlinearity is often known from first principles or can readily be determined by empirical means. Hence, improved control can be expected by utilizing a more accurate nonlinear model rather than an approximate linear model. In a few special situations, STC's based on nonlinear models have provided significant improvements over standard STC's based on linear models (Anbumani et al., 1981; Lachmann, 1982; Dochain and Bastin, 1984; Svoronos et al., 1981). However, the applicability of these techniques is limited to processes with specific nonlinearities such as polynomials in the manipulated input or bilinear products of output and input.

This paper presents a new self-tuning controller for multiple-input, multiple-output (MIMO) processes based on a very general class of nonlinear process models. The analysis is an extension of the modified nonlinear STC for single-input, single-output (SISO) systems that was previously developed by the authors (Agarwal and Seborg, 1985) to MIMO systems. The new MIMO STC algorithm provides several advantages over an alternative presented in an earlier paper (Agarwal and Seborg, 1986): it is applicable to a broader class of systems, it requires estimation of fewer parameters, and it allows different penalties on the elements of the output in the performance index.

Problem Statement

The problem is to derive a new self-tuning controller that minimizes a quadratic cost function objective based on a very general class of nonlinear models that can include arbitrary, known nonlinear functions of the old inputs and outputs as well as the products of these functions and any powers of the most recent inputs. It is assumed that the nonlinear MIMO process can be described adequately by a discrete-time model, which is linear in system parameters and allows for different numbers of inputs and outputs and different time delays between different input-output pairs. Such a model can be represented in the following general form:

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$$
\left.\begin{aligned}
y(t) &= \sum_{i=1}^{N} K(z^{-1})[B_i u^{(r_i)}(t) Y_i(t)] + d + C(z^{-1})\xi(t) \\
Y_i(t) &\equiv g_i[y_q(t + k_q - j)], j = 1, 2, 3, \ldots, \\
&\quad q = 1, 2, \ldots, n; \\
&\qquad u_p(t - h), h = 1, 2, 3, \ldots, \\
&\quad p = 1, 2, \ldots, m], \quad i = 1, 2, \ldots, N \\
u^{(r_i)}(t) &\equiv [u_1^{r_i}(t), u_2^{r_i}(t), \ldots, u_m^{r_i}(t)]^T; \quad i = 1, 2, \ldots, N \\
K(z^{-1}) &\equiv \text{diag}\,[z^{-k_1}, z^{-k_2}, \ldots, z^{-k_n}]
\end{aligned}\right\} \quad (1)
$$

where t is the sampling instant, $t = 1, 2, 3, \ldots$; N and r_i are known integers ($r_i \geq 0$); y is the $n \times 1$ measured output; u is the $m \times 1$ manipulated input; d is the $n \times 1$ unknown disturbance; $\{\xi(t)\}$ is a sequence of equally distributed random $n \times 1$ vectors, with zero means and unknown covariances, independent of previous inputs and outputs; g_i, $i = 1, 2, \ldots, N$, are known single-value d time-invariant functions involving known parameters; k_q, $q = 1, 2, \ldots, n$, is the known minimum time delay between any input and the qth output,

$$
k_q = \min\,[k_{qp}, p = 1, 2, \ldots, m] \geq 1, \quad q = 1, 2, \ldots, n \quad (2)
$$

where k_{qp} is the time delay between the qth output and the pth input, expressed as one plus the largest integer multiple of the sampling period smaller than or equal to the time delay; z^{-1} is the backward shift operator with the following properties

$$
\left.\begin{aligned}
z^{-j}\varphi(t) &= \varphi(t - j) \\
z^{-j}[V\varphi(t)\,\psi(t)] &= V\varphi(t - j)\psi(t - j) \\
z^{-j}\varphi(t_1|t_2) &= \varphi(t_1 - j|t_2 - j)
\end{aligned}\right\} \quad j = 1, 2, 3, \ldots \quad (3)
$$

where V is a constant matrix, φ is a vector of appropriate dimensions, ψ is a scalar, and t, t_1, t_2 are any discrete instants.

In Eq. 1, B_i, $i = 1, 2, \ldots, N$, are $n \times m$ matrices. It is assumed that those elements of B_i are zero that correspond to $K(z^{-1})[B_i u^{(r_i)}(t) Y_i(t)]$ being a function of $y_q(t + j)$, $j = 0, 1, \ldots$ in Eq. 1. That is, the righthand side of the process model in Eq. 1 is assumed to be independent of $y(t)$, $y(t + 1)$, ... Furthermore, C is an $n \times n$ diagonal polynomial matrix given by

$$
C(z^{-1}) = I + C_1 z^{-1} + \cdots + C_{nc} z^{-nc} \quad (4)
$$

with the assumption that all roots of det $C(z^{-1})$ lie inside the unit circle in the z plane.

Remark 1. The nonlinear system model in Eq. 1 is linear with respect to the system parameters B_i, $i = 1, 2, \ldots, N$; C; and d. Further, it does not require the numbers of inputs and outputs to be equal.

Remark 2. The model structure in Eq. 1 ensures that the predictions of future outputs will not require unknown values of the inputs. Such predictions are required for the STC algorithm developed in the next section. For any q, the prediction of y_q at time $t + k_q$ requires knowledge of unknown current inputs, and predictions at later times require knowledge of future inputs. Thus, $Y_i(t)$ is specified to be a function of y_q only at times prior to $t + k_q$, so that $Y_i(t)$ will depend only on known inputs.

Example. To illustrate the general nature of the nonlinear term in the process model in Eq. 1, consider a 2×2 system that has $k_1 = 4$, and $k_2 = 2$. Then, the first row of Eq. 1 representing $y_1(t)$ could conceivably be:

$$
\begin{aligned}
y_1(t) &= b_1 y_1(t - 1) + b_2 y_1(t - 2) + b_3 y_2(t - 3) \\
&\quad + b_4 y_2(t - 4) + b_5 u_1(t - 5) + b_6 u_2(t - 4) \\
&\quad + b_7 u_2(t - 5) + b_8 u_2^2(t - 4) + b_9 u_2^3(t - 4) \\
&\quad \cdot \exp\left[\frac{-1}{y_2(t - 3)}\right] + b_{10} y_1^{0.5}(t - 1) \\
&\quad \cdot \ln\,[u_1(t - 5)]u_2^{-1.7}(t - 5) + d_1 + \xi_1(t)
\end{aligned} \quad (5)
$$

On the righthand side of Eq. 5, notice that although the most recent y_1 is at time $t - 1$, the most recent y_2 occurs at time $t + k_2 - 1 - k_1$. This latter restriction is a consequence of Remark 2. Similarly, the most recent occurrence of any input can be no later than at instant $t - 4$, although the most recent u_1 occurs earlier, at instant $t - 5$, indicating that $k_{11} = 5$. In the nonlinear terms, the inputs at instant $t - 4$ can be raised only to integer powers, whereas the inputs at $t - 5$ can be subject to any single-valued nonlinearity.

To simplify notation define k such that for any $n \times 1$ vector x $\in n \times 1$ vector,

$$
x(t + k) \equiv [x_1(t + k_1), x_2(t + k_2), \ldots, x_n(t + k_n)]^T \quad (6)
$$

Then, the quadratic performance index to be considered can be written as

$$
\begin{aligned}
I = \mathcal{E}\,\{\|P'(z^{-1})y(t + k) + P''(z^{-1})\Delta y(t + k) \\
- R(z^{-1})w(t)\|^2 + \|Q'(z^{-1})u(t)\|^2\}
\end{aligned} \quad (7)
$$

where

$$
\Delta y(t) \equiv y(t) - y(t - 1) \quad (8)
$$

the notation $\|v\|^2 = v^T v$ is used, $\mathcal{E}$ is the expectation operator conditioned with respect to data up to time t, w is the $n \times 1$ set point vector, R is an $n \times n$ diagonal rational transfer function matrix, Q' is an $m \times m$ diagonal rational transfer function matrix, and P' and P'' are $n \times n$ diagonal polynomial matrices given by

$$
P'(z^{-1}) = I + P_1' z^{-1} + \cdots + P_{n_{P'}}' z^{-n_{P'}} \quad (9)
$$

$$
P''(z^{-1}) = P_0'' + P_1'' z^{-1} + \cdots + P_{n_{P''}}'' z^{-n_{P''}} \quad (10)
$$

Remark 3. This quadratic performance index is similar to those that have been used in previous STC studies (Seborg et al., 1986). However, the Δy term has been added to provide greater flexibility, following the suggestion of Waller and Gustafsson (1975) based on their experience with optimal control problems. This change was beneficial in our earlier analysis of nonlinear SISO systems (Agarwal and Seborg, 1986).

Although the user may find it more convenient to specify P' and P'' separately, these matrices can be combined such that the performance index in Eq. 7 becomes

$$
\begin{aligned}
I = \mathcal{E}\,\{\|P(z^{-1})y(t + k) \\
- R(z^{-1})w(t)\|^2 + \|Q'(z^{-1})u(t)\|^2\}
\end{aligned} \quad (11)
$$

where P is an $n \times n$ diagonal polynomial matrix given by

$$P(z^{-1}) = P_0 + P_1 z^{-1} + \cdots + P_{n_p} z^{-n_p} \quad (12)$$

A New Self-tuning Controller

The derivation of the new STC algorithm is similar in approach to that for the linear self-tuning regulator of Bayoumi et al. (1981). Define a diagonal polynomial matrix $F(z^{-1})$ as

$$F(z^{-1}) \equiv z[C(z^{-1})P(z^{-1}) - P_0] \quad (13)$$

Premultiplying Eq. 1 by $P_0 K^{-1}(z^{-1})$ and adding to it Eq. 13 postmultiplied by $K^{-1}(z^{-1})y(t)$, gives

$$C(z^{-1})P(z^{-1})y(t+k) = F(z^{-1})y(t+k-1) + \sum_{i=1}^{N} \cdot [P_0 B_i u^{(r_i)}(t) Y_i(t)] + P_0 d + C(z^{-1})P_0 \xi(t+k) \quad (14)$$

Define a k step predictor, $y^*(t+k|t)$, such that

$$P(z^{-1})y(t+k) = P(z^{-1})y^*(t+k|t) + P_0 \xi(t+k) \quad (15)$$

where

$$y^*(t+k|t) \equiv [y_1^*(t+k_1|t), \cdot y_2^*(t+k_2|t), \ldots, y_n^*(t+k_n|t)]^{\mathrm{T}} \quad (16)$$

Combining Eqs. 14 and 15 gives

$$C(z^{-1})P(z^{-1})y^*(t+k|t) = F(z^{-1})y(t+k-1) + \sum_{i=1}^{N} [P_0 B_i u^{(r_i)}(t) Y_i(t)] + P_0 d \quad (17)$$

The k step predictor in Eq. 17 is dependent on future values of y, Y_i. In order to obtain a predictor based on only the current and past information, the future values of y, Y_i in Eq. 17 are replaced by their predictions made at time t. Thus Eq. 17 becomes

$$C(z^{-1})P(z^{-1})y^*(t+k|t) = F(z^{-1})\tilde{y}(t+k-1) + \sum_{i=1}^{N} [P_0 B_i u^{(r_i)}(t) \tilde{Y}_i(t)] + P_0 d \quad (18)$$

where $\tilde{y}(t+1), \tilde{y}(t+2), \ldots, \tilde{y}(t+k-1)$ are defined as the future predictions of y based on information available at time t, $\tilde{Y}_i$ is computed using $\tilde{y}$ identically as Y_i is computed using y, and

$$\tilde{y}(j) = y(j), \quad j \leq t \quad (19)$$

The predictions $\tilde{y}(t+1), \tilde{y}(t+2), \ldots, \tilde{y}(t+k-1)$ are calculated recursively from the deterministic part of Eq. 1, upon multiplication by z^{+j}, as

$$\tilde{y}(t+j) = \sum_{i=1}^{N} K(z^{-1})[B_i u^{(r_i)}(t+j)\tilde{Y}_i(t+j)] + d, \quad j = 1, 2, \ldots, k-1 \quad (20)$$

where the notation $j = k - 1$ implies $j = k_q - 1$ for the qth row of Eq. 20. The performance index in Eq. 11 can be written, using Eq. 15, as

$$I = \mathscr{E}\{\|P(z^{-1})y^*(t+k|t) + P_0\xi(t+k) - R(z^{-1})w(t)\|^2 + \|Q'(z^{-1})u(t)\|^2\} \quad (21)$$

Since $P_0\xi(t+k)$ is not correlated with the other terms on the righthand side of Eq. 21, the performance index becomes

$$I = \|P(z^{-1})y^*(t+k|t) - R(z^{-1})w(t)\|^2 + \|Q'(z^{-1})u(t)\|^2 + \mathscr{E}\{\|P_0\xi(t+k)\|^2\} \quad (22)$$

Here, an expectation operator acting on $y^*(t+k|t)$ is not required since it is a deterministic, known quantity. The minimization of I can now be considered as a deterministic optimization problem. A necessary condition for an optimum is

$$\frac{\partial I}{\partial u(t)} = 0 \quad (23)$$

or

$$2\left\{\frac{\partial[P(z^{-1})y^*(t+k|t)]}{\partial u(t)}\right\}^{\mathrm{T}} [P(z^{-1})y^*(t+k|t) - R(z^{-1})w(t)] + 2[(Q'(0))]^{\mathrm{T}} Q'(z^{-1})u(t) = 0 \quad (24)$$

Differentiating Eq. 18 with respect to $u(t)$, and noting that $C(0) = I$, gives

$$\frac{\partial[P(z^{-1})y^*(t+k|t)]}{\partial u(t)} = \sum_{\substack{i=1 \\ r_i \neq 0}}^{N} [P_0 B_i r_i U^{(r_i-1)}(t)\tilde{Y}_i(t)] \quad (25)$$

where $U^{(r_i-1)}$ is an $m \times m$ diagonal matrix with the elements $u_1^{(r_i-1)}, u_2^{(r_i-1)}, \ldots, u_m^{(r_i-1)}$ comprising the diagonal. Substituting Eqs. 18 and 25 in Eq. 24 gives the control law

$$\left\{\sum_{\substack{i=1 \\ r_i \neq 0}}^{N} [P_0 B_i r_i U^{(r_i-1)}(t)\tilde{Y}_i(t)]\right\}^{\mathrm{T}} \left\{C^{-1}(z^{-1})F(z^{-1})\tilde{y}(t+k-1) + \sum_{i=1}^{N} C^{-1}(z^{-1})[P_0 B_i u^{(r_i)}(t)\tilde{Y}_i(t)] + C^{-1}(z^{-1})P_0 d - R(z^{-1})w(t)\right\} + [Q'(0)]^{\mathrm{T}} Q'(z^{-1})u(t) = 0 \quad (26)$$

This can be rearranged as

$$\left.\begin{aligned}
&\sum_{\substack{i=1 \\ r_i \neq 0}}^{N} [P_0 B_i u^{(r_i)}(t)\tilde{Y}_i(t)] + \mu + C(z^{-1}) \\
&\quad \cdot \left(\left\{\sum_{\substack{i=1 \\ r_i \neq 0}}^{N} [P_0 B_i r_i U^{(r_i-1)}(t)\tilde{Y}_i(t)]\right\}^{\mathrm{T}}\right)^{-1} \\
&\quad \cdot [Q'(0)]^{\mathrm{T}} Q'(z^{-1})u(t) = 0 \\
&\quad \mu \equiv F(z^{-1})\tilde{y}(t+k-1) + \sum_{\substack{i=1 \\ r_i = 0}}^{N} [P_0 B_i u^{(r_i)}(t)\tilde{Y}_i(t)] \\
&\quad + P_0 d - C(z^{-1})R(z^{-1})w(t)
\end{aligned}\right\} \quad (27)$$

Here μ is the $n \times 1$ vector which is the sum of $u(t)$-independent terms on the lefthand side of Eq. 26.

The control law in Eq. 27 is in the form of polynomials involving elements of $u(t)$. When the parameters of the model in Eq. 1 are either known or estimated, $F(z^{-1})$ can be evaulated using Eq. 13, and $\tilde{y}$ using Eq. 20, and the control law in Eq. 27 can be solved using standard numerical methods. Only the real solutions that satisfy any given constraints on u are relevant, and in case of more than one relevant solution, the one closest to the previous control action, $u(t-1)$, can be selected for $u(t)$. In case there is no relevant solution, the set point, $w(t)$, can be altered temporarily as suggested by Lachmann (1982).

If the system model in Eq. 1 is linear with respect to the most recent control input (i.e., $r_i \leq 1, i = 1, 2, \ldots, N$), then the control law in Eq. 27 has the explicit representation:

$$\left.\begin{aligned} u(t) &= -\{\Psi + C(z^{-1})(\Psi^T)^{-1}[Q'(0)]^T Q'(z^{-1})\}^{-1}\mu \\ \Psi &= \sum_{\substack{i=1 \\ r_i \neq 0}}^{N} [P_0 B_i \tilde{Y}_i(t)] \end{aligned}\right\} \quad (28)$$

provided that the inverses exist.

Remark 4. Since $Q'(z^{-1})$, which is defined in Eq. 7, appears in the control law in Eqs. 27 and 28 only in conjunction with $Q'(0)$, the user need only specify

$$Q(z^{-1}) = [Q'(0)]^T Q'(z^{-1}) \quad (29)$$

On-line parameter estimation

If the model parameters are not known, input-output data can be used for on-line estimation of parameters B_i, $i = 1, 2, \ldots, N$; $C(z^{-1})$; and d. Adding Eqs. 15 and 17, and premultiplying by $K(z^{-1})$, gives

$$\begin{aligned} P(z^{-1})y(t) = F(z^{-1})y(t-1) &+ \sum_{i=1}^{N} K(z^{-1}) \\ &\cdot [P_0 B_i u^{(r_i)}(t) Y_i(t)] + P_0 d \\ &+ z[I - C(z^{-1})][P(z^{-1}) \\ &\cdot y^*(t-1|t-k-1)] \\ &+ P_0 \xi(t) \end{aligned} \quad (30)$$

Substituting for $F(z^{-1})$ from Eq. 13 into Eq. 30, yields

$$\begin{aligned} P_0 y(t) = \sum_{i=1}^{N} K(z^{-1})[P_0 B_i u^{(r_i)}(t) Y_i(t)] + P_0 d \\ + z[C(z^{-1}) - I][P(z^{-1})y(t-1) \\ - P(z^{-1})y^*(t-1|t-k-1)] + P_0 \xi(t) \end{aligned} \quad (31)$$

Since $P_0\xi(t)$ is uncorrelated with all other terms on the righthand side of Eq. 31, the extended least-squares method can be employed in the standard manner to estimate the unknown parameters. Then $F(z^{-1})$ is obtained using the estimate of $C(z^{-1})$ and Eq. 13. These estimated parameters are used in Eqs. 27 and 20 for implementation of the control law. The number of parameters that need to be estimated for this algorithm is equal to the number of unknown parameters in the system model of Eq. 1.

Table 1. Nominal Operating Condition for Distillation Column

Symbol	Description	Value
X_D	Distillate mol frac.	0.040
X_B	Bottoms mol frac.	0.725
L	Reflux flow rate	0.01342 kg/s
P_s	Reboiler steam press.	1.9785 atm
F	Feed flow rate	0.01368 kg/s
X_{F_n}	n-Butanol in feed	0.380
X_{F_s}	s-Butanol in feed	0.340
X_{F_t}	t-Butanol in feed	0.280

SI conversion: kPa = atm × 101.325

Table 2. Open-loop Process Gains for Distillation Column

ΔL kg/s	ΔP_s atm	$X_D - L$ (kg/s)$^{-1}$	$X_D - P_s$ atm^{-1}	$X_B - L$ (kg/s)$^{-1}$	$X_B - P_s$ atm^{-1}
+0.01	0.0	−2,600	—	−16,600	—
−0.01	0.0	−13,200	—	−6,100	—
0	+0.2	—	0.0825	—	0.395
0	−0.2	—	0.0660	—	0.420

SI conversion: kPa = atm × 101.325

Simulation Results

A pilot-scale distillation column at UCSB was simulated using a dynamic model developed by Wong (1985). The 6 in. dia. column utilizes 12 sieve trays to distill a ternary mixture consisting of normal, secondary, and tertiary butanol. The mole fractions of n-butanol in the distillate and bottoms streams, X_D and X_B, are controlled by manipulating the reflux flow rate, L,

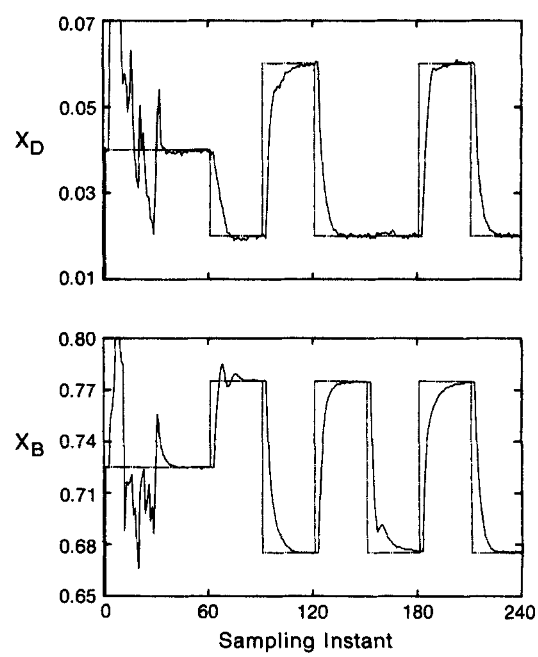

Figure 1. Performance of nonlinear STC.

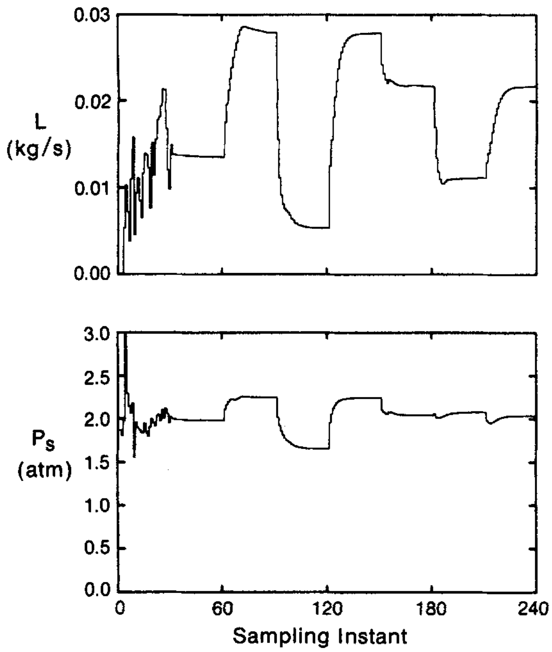

Figure 2. Manipulated variables for Figure 1.

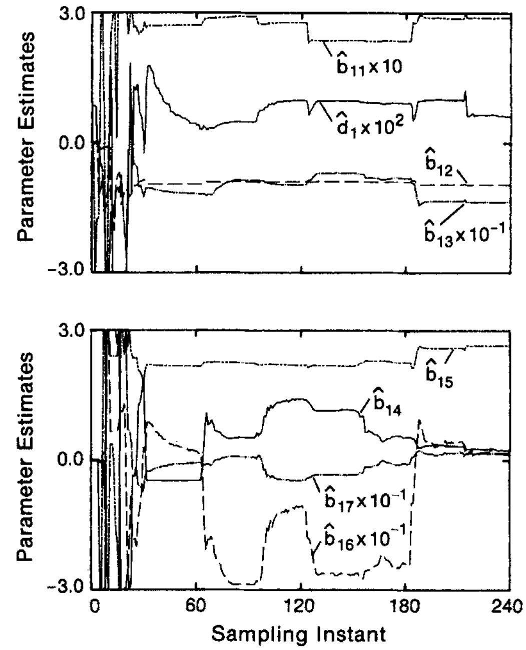

Figure 3. Estimates for y_1 parameters for Figure 1.

and the reboiler steam pressure, P_s. The process model is very nonlinear (Wong, 1985) and in the form,

$$\left.\begin{aligned} 160\dot{y}_1(t') &= -y_1(t') + [-1.36 - 19.3y_1'(t') \\ &\quad + 0.0951y_2'(t')]u_1(t' - 650) \\ &\quad + [3.74 + 0.584y_1'(t') \\ &\quad + 3.50y_2'(t')]u_2(t' - 690) \\ 200\dot{y}_2(t') &= -y_2(t') + [-0.125 - 0.237y_1'(t') \\ &\quad + 0.636y_2'(t')]u_1(t' - 640) \\ &\quad + [1.11 - 2.04y_1'(t') \\ &\quad - 0.0229y_2'(t')]u_2(t' - 650) \\ y_1'(t') &\equiv \frac{y_1(t')}{24 - y_1(t')} \\ y_2'(t') &\equiv \frac{y_2(t')}{1 + y_2(t')} \end{aligned}\right\} \quad (32)$$

where t' is continuous time in seconds; y_1 and y_2 are the controlled outputs defined as normalized deviations of the n-butanol mole fractions in the distillate and bottoms streams, respectively; and u_1 and u_2 are the manipulated inputs defined as normalized deviations of the reflux flow and the reboiler steam pressure, respectively. Table 1 shows the relevant variables at the nominal operating condition.

Open-loop process gains are shown in Table 2 for changes in both inputs. The gains were determined as the change in output from the nominal condition divided by the change in input from the nominal condition. The first element of the relative gain array for these open-loop changes is 1.35. Thus, the simulated process shows some interaction between the two outputs.

For the simulation study, the process response was calculated using the fourth-order Runge-Kutta numerical integration method with a time step of 2 s. In view of the 0.1% accuracy in the gas chromatograph composition measurements reported by Wong (1985), both the outputs were contaminated by zero-mean Gaussian noise with a standard deviation of 0.03%. The outputs were sampled once every 280 s as in the study by Wong. The reflux flow was constrained between 0 and 0.03 kg/s, and the reboiler steam pressure was allowed to vary between 0 and 3 atm (0–303.9 kPa).

The simulated process was controlled using the new nonlinear

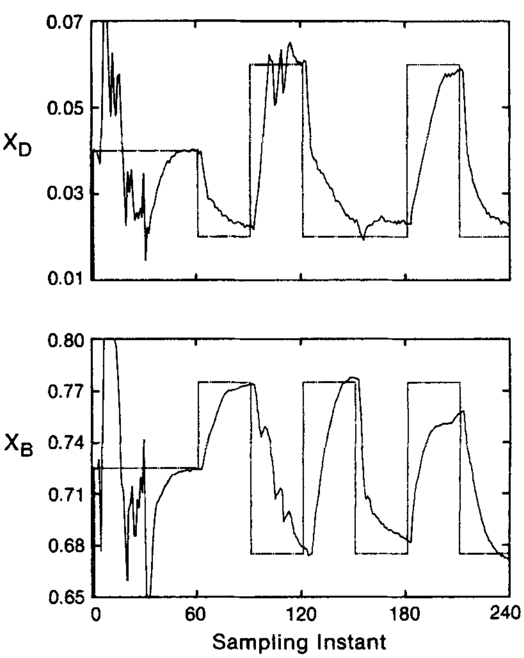

Figure 4. Performance of linear STC.

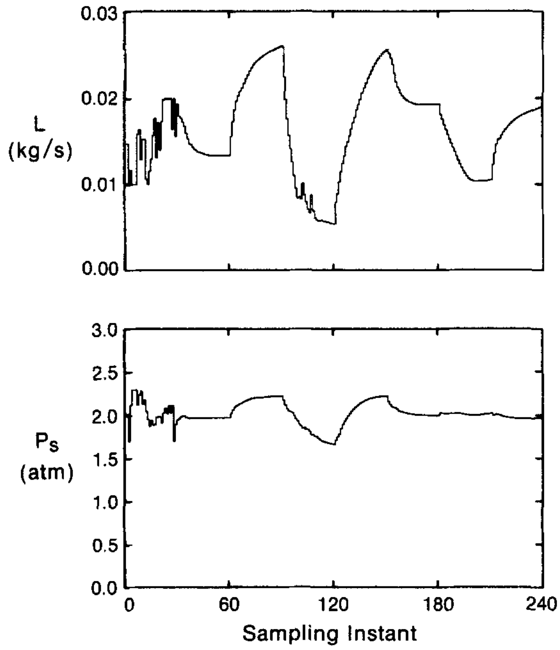

Figure 5. Manipulated variables for Figure 4.

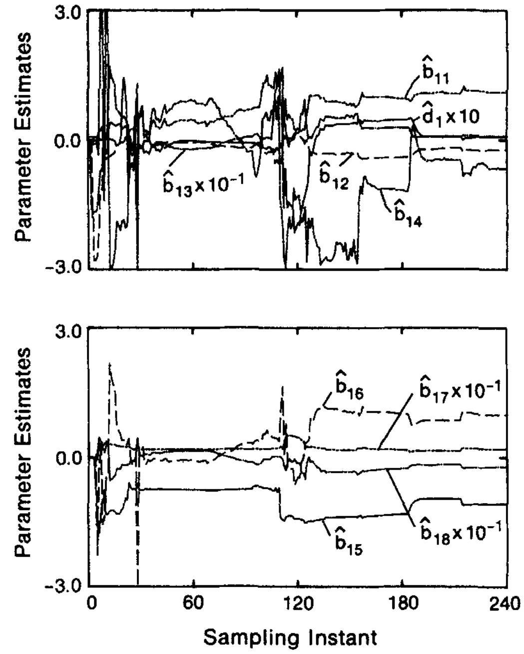

Figure 6. Estimates for y_1 parameters for Figure 4.

STC based on the nonlinear discrete-time model:

$$\left.\begin{aligned} y_1(t) &= b_{11}y_1(t-1) + [b_{12} + b_{13}y_1'(t-1) \\ &\quad + b_{14}y_2'(t-1)]u_1(t-3) \\ &\quad + [b_{15} + b_{16}y_1'(t-1) + b_{17}y_2'(t-1)] \\ &\quad \cdot u_2(t-3) + d_1 + \xi_1(t) \\ y_2(t) &= b_{21}y_2(t-1) + [b_{22} + b_{23}y_1'(t-1) \\ &\quad + b_{24}y_2'(t-1)]u_1(t-3) \\ &\quad + [b_{25} + b_{26}y_1'(t-1) + b_{27}y_2'(t-1)] \\ &\quad \cdot u_2(t-3) + d_2 + \xi_2(t) \\ y_1'(t) &= \frac{y_1(t)}{24 - y_1(t)} \\ y_2'(t) &= \frac{y_2(t)}{1 + y_2(t)} \end{aligned}\right\} \tag{33}$$

The performance of the new STC was compared with the performance of a linear STC that was based on the model:

$$\left.\begin{aligned} y_1(t) &= b_{11}y_1(t-1) + b_{12}y_1(t-2) + b_{13}y_2(t-1) \\ &\quad + b_{14}y_2(t-2) + b_{15}u_1(t-3) + b_{16}u_1(t-4) \\ &\quad + b_{17}u_2(t-3) + b_{18}u_2(t-4) + d_1 + \xi_1(t) \\ y_2(t) &= b_{21}y_1(t-1) + b_{22}y_1(t-2) + b_{23}y_2(t-1) \\ &\quad + b_{24}y_2(t-2) + b_{25}u_1(t-3) + b_{26}u_1(t-4) \\ &\quad + b_{27}u_2(t-3) + b_{28}u_2(t-4) + d_2 + \xi_2(t) \end{aligned}\right\} \tag{34}$$

In both the above models, normalized deviation variables were used for the inputs and the outputs.

All parameters were initialized to zero and estimated using the recursive least-squares method. The UDU covariance factorization method of Thornton and Bierman (1978) was employed with a forgetting factor of 0.98. The diagonal elements of the covariance matrix were initialized to 10^6, and were subsequently reset to this value whenever the magnitude of the estimation error exceeded 0.05 for y_1 and 0.01 for y_2. A larger estimation error was allowed for y_1 because the relative magnitude of the measurement noise was larger due to the smaller nominal value of this output. The STC's were initialized at the beginning of each run; but in order to facilitate estimation initially, for the

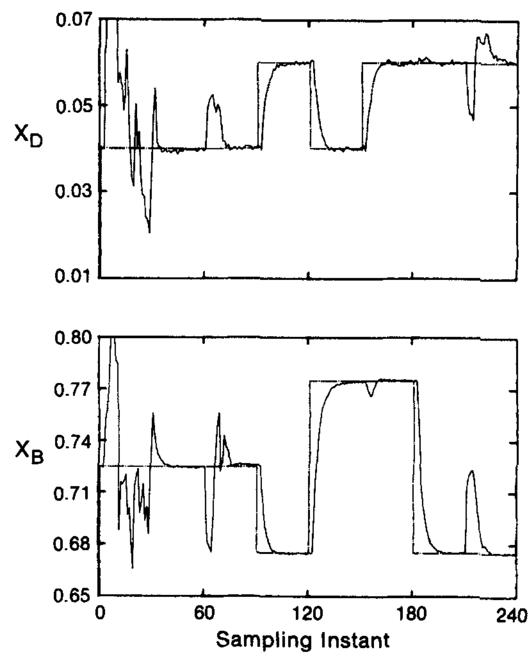

Figure 7. Performance of nonlinear STC for load.

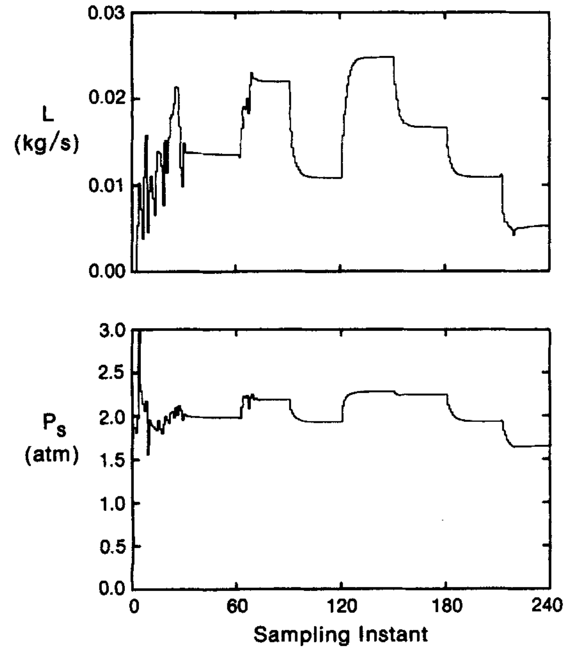

Figure 8. Manipulated variables for Figure 7.

first 30 sampling instants, pseudorandom binary sequences (PRBS) of magnitude 0.003 kg/s and 0.05 atm (5.07 kPa) were added to the reflux flow and the reboiler steam pressure inputs, respectively, during closed-loop operation. During the initial estimation period of 60 sampling instants, the linear STC required narrower constraints on the inputs in order to attain parameter convergence: the reflux flow had to be limited between 0.01 and 0.02 kg/s, and the reboiler steam pressure between 1.7 and 2.3 atm (172.2–233 kPa). The nonlinear STC

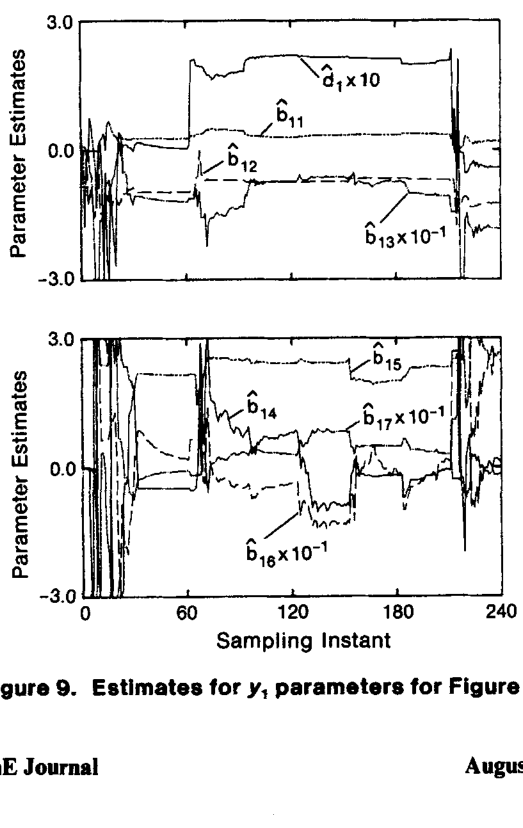

Figure 9. Estimates for y_1 parameters for Figure 7.

converged without any special input constraints during this period.

The process was initialized to the nominal condition and set point changes were specified between 0.020 and 0.060 for X_D and between 0.675 and 0.775 for X_B. The set point changes included simultaneous changes in both set points in the same direction and in opposite directions in order to provide large changes in the operating conditions. The performance of the nonlinear STC with $\boldsymbol{P} = (1 - 0.7z^{-1})\boldsymbol{I}$, $\boldsymbol{Q} = 0.001(1 - z^{-1})\boldsymbol{I}$, and $\boldsymbol{R} = 0.3\boldsymbol{I}$ is shown in Figures 1–3. It can be seen that the outputs track the set points satisfactorily. Figure 3 shows that the estimate of bias d_1 is nonzero even though the load is zero. This is due to the approximation involved in obtaining the discrete-time model in Eq. 33 from the continuous-time process model in Eq. 32. By contrast, Figures 4–6 show the performance of the linear STC with $\boldsymbol{P} = (1 - 0.85z^{-1})\boldsymbol{I}$, $\boldsymbol{Q} = 0.01(1 - z^{-1})\boldsymbol{I}$, and $\boldsymbol{R} = 0.15\boldsymbol{I}$. Even after considerable detuning, the performance of the best-tuned linear STC is seen to be inferior to that of the new nonlinear STC. The parameter estimates for the linear STC shown in Figure 6 exhibit more changes for different operating conditions than do the estimates for the nonlinear STC shown in Figure 3. The parameter estimates corresponding to y_2 showed similar behavior. A linear STC with parameters b_{12}, b_{14}, b_{16}, b_{18}, b_{22}, b_{24}, b_{26}, and b_{28} set to zero in Eq. 34 also performed poorly.

Load changes were also simulated using the feed flow rate as the load variable. Since the process model in Eq. 32 does not include load variables, the normalized deviation of the feed flow rate was passed through two first-order transfer functions, with gains equal to 0.06 and −0.24 and time constants equal to 160 and 200 s, respectively, and added to the process outputs y_1 and y_2 calculated from Eq. 32. The feed flow rate was increased by 20% at sampling instant 60, and was restored to the nominal value at instant 210. Figures 7–9 show the performance of the nonlinear STC with the same tuning as in Figure 1. In Figure 7, the set point for X_D has not been lowered to 0.02, since this causes the reflux flow to exceed the high constraint. It is seen that the nonlinear STC responds well to the load changes and preserves good set point tracking at the new load condition. Figure 9 shows that the estimate of the bias d_1 changes considerably whenever the load changes, in comparison to the relatively constant estimate of d_1 in Figure 3.

The linear STC with the same tuning as in Figure 4 became unstable upon introduction of the load. Thus the nonlinear STC was superior to the linear STC for both load and set point changes.

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Notation

- b_i, b_{ij} = model parameters
- $\boldsymbol{B}_i$ = $n \times m$ matrix of model parameters
- $\boldsymbol{C}$ = $n \times n$ polynomial matrix of model parameters
- $\boldsymbol{C}_i$ = $n \times n$ matrix of ith coefficients of $\boldsymbol{C}$
- $\boldsymbol{d}$ = $n \times 1$ vector of unknown disturbances
- d_i = ith element of vector $\boldsymbol{d}$
- F = feed flow rate
- $\boldsymbol{F}$ = $n \times n$ polynomial matrix, Eq. 13
- g_i = known nonlinear function
- I = scalar cost function

I = identity matrix
k = time delay shift for an $n \times 1$ vector, Eq. 6
k_q = minimum time delay for qth output
k_{qp} = time delay between qth output and pth input
K = $n \times n$ matrix of minimum time delays
L = reflux flow rate
m = number of inputs
n = number of outputs
n_C = order of polynomial matrix C
$n_P, n_{P'}, n_{P''}$ = orders of polynomial matrices P, P', P''
N = known integer
P_s = reboiler steam pressure
P, P', P'' = $n \times n$ polynomial matrices of output penalties
P_i, P_i', P_i'' = $n \times n$ matrices of ith coefficients of P, P', P''
Q, Q' = $m \times m$ matrices of input penalty
r_i = known integers
R = $n \times n$ matrix of set point filter
t = discrete time
t' = continuous time
u = $m \times 1$ vector of manipulated inputs
$u^{(r_i)}$ = $m \times 1$ vector of manipulated inputs raised to power of r_i
u_p = pth manipulated input
$U^{(r_i)}$ = $m \times m$ diagonal matrix of manipulated inputs raised to power r_i
V = a constant matrix
w = $n \times 1$ vector of set points
x = $n \times 1$ vector
x_i = ith element of vector x
X_B = n-butanol mole fraction in bottoms
X_D = n-butanol mole fraction in distillate
$X_{F_n}, X_{F_s}, X_{F_t}$ = n-, s-, t-butanol mole fractions in feed
y = $n \times 1$ vector of measured outputs
y_q = qth measured output
y_q' = functions of y_q, Eqs. 32, 33
Y_i = scalar nonlinear function of inputs and outputs
z^{-1} = backward shift operator

Greek letters

Δ = incremental change operator
$\mathcal{E}$ = expectation operator
ξ = $n \times 1$ vector of random noise
ξ_i = ith element of vector ξ
μ = $n \times 1$ vector, Eq. 27
Ψ = $n \times m$ matrix, Eq. 28

Superscripts

T = transpose
* = predicted value, Eq. 15
~ = predicted value, Eqs. 19, 20

Subscripts

p = pth input
q = qth output

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